Constitutive Modeling of Discontinuous Carbon Fiber Polymer Composites

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Overview

• Material modeling
  – Micro-mechanics-based damage model
  – Fracture mechanics-based damage model
  – Delamination Model
  – Preform Modeling

• Progressive Crush Experiments

• Future Work
http://www-cms.ornl.gov/composites/

- Select ‘Reports’ for documentation

Objectives:

The objective of the project is to develop analytical and numerical tools that efficiently predict the behavior of carbon-fiber based composites in vehicular crashworthiness simulations. This project focuses specifically on molded polymeric matrix composites, and considers loading conditions and strain rates that arise in vehicular impacts.

While the short and intermediate goals are to provide approaches and numerical methods to simulate automotive components during impact events using the composite mechanical properties, the long term goal strives to...
Micromechanics-Based Constitutive Model of Discontinuous Fiber Composite Materials

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Modeling of Discontinuous Fiber Composites

Objective

- Develop analytical and numerical tools for predicting mechanical response of discontinuous carbon fiber composites subjected to impact loading.

Accomplishments

- Developed constitutive models based on micro-mechanical formulation and combination of micro- and macro-mechanical damage criteria.
- Incorporated developed models with probabilistic micro-mechanics for evolutionary damage in composite materials.
- Investigated effect of weakened interface (partial debonding) by employing an equivalent, transversely isotropic inclusion.
- Extended developed model to include crack effects on mechanical response of composite materials.
Accomplishments (Continued)

- Developed micromechanics-based inter-fiber interaction formulation for modeling of composites with high fiber volume fraction.

- Implemented models into finite element code DYNA3D to perform impact simulation of composite materials.

- Performed numerical simulations for Biaxial Test of Cruciform Shaped Composite Specimen and Four-point Bend Test in order to evaluate ability of developed composite model to predict experimentally obtained response.

- Performed parametric studies to evaluate constitutive model sensitivity to Weibull parameter $S_o$. 
Publications


Outline

- Micromechanics-based Constitutive material model
- Progressive damage model
- Crack and inter-fiber interaction effects
- Finite element implementation for impact simulation
- Iterative algorithm for progressive damage model
- Biaxial impact simulation of cruciform shaped composite specimen
- Four-point bend simulation
- Summary and future directions
Micromechanics and Equivalence Principle

\[ \sigma^0 \text{ or } \varepsilon^0 \]

Heterogeneous composites

Micromechanics Based modeling

Equivalent homogeneous media with appropriately defined properties
Composites with Aligned Discontinuous Fibers

† Effective elastic moduli of multi-phase composites containing randomly located, aligned elastic ellipsoids (see publication #2)

\[ C_\ast = C_0 \cdot \left\{ I + B \cdot (I - S \cdot B)^{-1} \right\} \]

† Total stress at any point \( \mathbf{x} \) in the matrix

\[ \sigma(\mathbf{x}) = \sigma^o + \sigma'(\mathbf{x}) \]

in which \( \sigma^o \) and \( \sigma' \) are defined as

\[ \sigma^o \equiv C_0 : \epsilon^o \]

\[ \sigma'(\mathbf{x}) \equiv C_0 : \int_V G(\mathbf{x} - \mathbf{x}') : \epsilon_1^i(\mathbf{x}') \, d\mathbf{x}' + C_0 : \int_V G(\mathbf{x} - \mathbf{x}') : \epsilon_2^s(\mathbf{x}') \, d\mathbf{x}' \]
Composites with Aligned Discontinuous Fibers

† Ensemble-averaged stress norm for any matrix point $\mathbf{x}$

$$
\langle H \rangle_m(\mathbf{x}) \equiv H^o + \int_{|\mathbf{x}-\mathbf{x}^{(1)}_1|>a} \left\{ H(\mathbf{x}|\mathbf{x}^{(1)}_1) - H^o \right\} P(\mathbf{x}^{(1)}_1) \, d\mathbf{x}^{(1)}_1 \\
+ \int_{|\mathbf{x}-\mathbf{x}^{(1)}_2|>a} \left\{ H(\mathbf{x}|\mathbf{x}^{(1)}_2) - H^o \right\} P(\mathbf{x}^{(1)}_2) \, d\mathbf{x}^{(1)}_2 + \ldots
$$

† Effective yield function for fiber-reinforced composites

$$
\bar{F} = (1 - \phi_1)^2 \mathbf{\bar{\sigma}} : \mathbf{\bar{T}} : \bar{\mathbf{\sigma}} - K^2(\bar{\varepsilon}^p)
$$

where the isotropic hardening function $K(\varepsilon^p)$ is defined as

$$
K(\varepsilon^p) = \sqrt{\frac{2}{3}} \left\{ \sigma_y + h(\varepsilon^p)^\eta \right\}
$$
Randomly Oriented, Discontinuous Fibers

† Orientational averaging process (see publication #1)

\[ \langle \sigma \rangle \equiv \int_0^{2\pi} \int_0^{\pi/2} (\cdot) P(\theta, \phi) \sin \theta \, d\theta \, d\phi \]

† Governing equations for randomly oriented fiber-reinforced composites

\[ \begin{align*}
\langle \sigma \rangle & = C_0 : [\langle \bar{\epsilon} \rangle - \sum_{r=1}^{2} \phi_r \langle \epsilon^*_r \rangle] \\
\langle \bar{\epsilon} \rangle & = \chi : \epsilon^o \\
\langle \epsilon^*_r \rangle & = -\Omega_r : \epsilon^o
\end{align*} \]
Progressive Damage Model

The initial state

The damaged state

† A schematic of aligned fiber composites subjected to uniaxial tension.
See Figure A.
Figure A. A schematic of the equivalence between a partially debonded fiber and an equivalent perfectly bonded, transversely isotropic inclusion.
Progressive Damage Model

† Weibull probabilistic distribution function for fiber debonding (damage)

\[ P_d[(\bar{\sigma}_m)_1] = 1 - \exp \left[ - \left( \frac{(\bar{\sigma}_m)_1}{S_o} \right)^M \right] \]

Current debonded (damaged) fiber volume fraction

\[ \phi_2 = \phi P_d[(\bar{\sigma}_m)_1] = \phi \left\{ 1 - \exp \left[ - \left( \frac{(\bar{\sigma}_m)_1}{S_o} \right)^M \right] \right\} \]
Penny-Shaped Cracks

Penny-shaped cracks are regarded as the limiting case of oblate spheroidal voids with aspect ratio \( \alpha \to 0 \).

Various cracks with different crack size and orientation can be included into the constitutive model by adding the corresponding inclusion phases.

Penny-shaped cracks are assumed to remain at the same crack density during the deformations.

Effective elastic moduli for multi-phase composites containing various cracks is derived as (see publication #3):

\[
C_n = C_o \cdot \left[ I + \sum_{r=1}^{n} \phi_r (A_r + S_r)^{-1} \cdot [I - \phi_r S_r \cdot (A_r + S_r)^{-1}]^{-1} \right]
\]

If \( \alpha > 1 \); prolate spheroidal inclusion
\( \alpha = 1 \); spherical inclusion
\( \alpha < 1 \); oblate spheroidal inclusion

Spheroidal inclusion
Effect of Cracks on Mechanical Behavior of Composite

- high crack density without debonding
- low crack density without debonding
- high crack density with debonding
- low crack density with debonding
Inter-Fiber Interaction Formulation

Local matrix point $x_m$ collects stress perturbations due to surrounding fibers with pairwise inter-fiber interaction.

† Approximate eigenstrain accounting for pairwise fiber interaction for multi-phase composites

$$< \varepsilon^* > = \Gamma : \varepsilon^o$$

† Effective elastic moduli for multi-phase composites considering inter-fiber interaction (The corresponding paper is in preparation.)

$$C^e = C_o \cdot \left\{ I - \sum_{r=1}^{n} \phi_r \Gamma \cdot (A_r - S_r + \phi_r S_r \cdot \Gamma)^{-1} \right\}$$
Inter-Fiber Interaction Formulation

Inter-fiber interactions significantly affect the overall composite moduli when the contrast ratio is high (or low).
Finite Element Implementation for Impact Simulation

- Model is implemented into finite element code DYNA3D to perform impact simulation of composite materials.
- Model uses strain driven algorithm to link with displacement based finite element code DYNA3D.
- Micromechanical iterative algorithm is used for modeling progressive damage.
Iterative Algorithm for Progressive Damage Model

See Box 1.
**Box 1 Iterative algorithm for progressive damage model**

(i) Compute (see Probabilistic approach for progressive damage):

Damaged (Partially debonded) fiber material properties: $k_3, l_3, m_3, n_3, p_3$

(ii) Estimate model parameters (refer to Parameter estimation algorithm):

Plastic material parameters: $\sigma_y, \beta, g_n$  Weibull parameters: $S_o, M$

(iii) Initialize: set $z = 0$; $(\phi_1)_{n+1}^{(0)} = (\phi_1)_n$, $(\phi_2)_{n+1}^{(0)} = (\phi_2)_n$

(iv) Compute (see Constitutive material model formulations):

Effective moduli: $(\kappa_0)_{n+1}^{(2)}$, $(\mu_0)_{n+1}^{(2)}$, $(E_0)_{n+1}^{(2)}$, $(\nu_0)_{n+1}^{(2)}$:

Internal stresses of fibers: $\lbrack [\sigma_m]_i \rbrack_{n+1}^{(e)}$

(v) Compute the Weibull probability distribution function:

$$P_d \{ \lbrack [\sigma_m]_i \rbrack_{n+1}^{(e)} \} = 1 - \exp \left[ - \left( \frac{\lbrack [\sigma_m]_i \rbrack_{n+1}^{(e)}}{S_o} \right)^M \right]$$

(vi) Compute volume fractions:

$$(\phi_2)_{n+1}^{(e)} = \phi P_d \{ \lbrack [\sigma_m]_i \rbrack_{n+1}^{(e)} \} = \phi \left\{ 1 - \exp \left[ - \left( \frac{\lbrack [\sigma_m]_i \rbrack_{n+1}^{(e)}}{S_o} \right)^M \right] \right\},$$

$$(\phi_1)_{n+1}^{(e)} = 1 - (\phi_2)_{n+1}^{(e)}$$

(vii) Perform convergence check:

If

$$\left| \frac{(\phi_1)_{n+1}^{(e)} - (\phi_1)_{n+1}^{(e-1)}}{(\phi_1)_{n+1}^{(e-1)}} \right| \leq TOL \ (e.g., \ 10^{-8})$$

then update quantities in (iv), (vi)

$$(\sigma_1)_{n+1}^{(e)} = \lbrack [\sigma_m]_i \rbrack_{n+1}^{(e)} \ (i = 1, 2, 3);$$

$$(\phi_1)_{n+1} = (\phi_1)_{n+1}^{(e)}; \ (\phi_2)_{n+1} = (\phi_2)_{n+1}^{(e)}; \ \ \text{GO TO (viii) in Box 2.}$$

Otherwise: SET $z = z + 1$; GO TO (iv).
Return Mapping Algorithm

See Box 2.
Box 2. 3-D return mapping algorithm

(viii) 3-D return mapping algorithm (based on strain driven algorithm):

(a) Initialize: set \( l = 0 \); \( (\mathbf{e}_n^{(0)}) = \mathbf{e}_n^p; \ (\mathbf{e}_n^{(0)}) = \mathbf{e}_n^p \) for local Newton iteration

(b) Compute: \( \bar{\mathbf{e}}_n = \mathbf{C}_n : [\mathbf{e}_n - \mathbf{e}_n^p] \)

(c) Compute the elastic predictor:

Trial elastic state: \( \mathbf{e}_{n+1}^r = \mathbf{e}_n + \mathbf{C}_n : \Delta \mathbf{e}_{n+1} \)

Compute: \( \mathbf{e}_{n+1} - \mathbf{e}_n + \Delta \mathbf{e}_{n+1}; \)

\[ F_{n+1}^{\text{cr}}(\mathbf{e}_{n+1}^r, \mathbf{e}_n^p) = (1 - (\phi_1)_n)\mathbf{e}_{n+1}^r : \mathbf{T}_{n+1} : \mathbf{e}_{n+1}^r - K^2(\mathbf{e}_n^p) \]

(d) Check whether plastic loading is active:

If \( F_{n+1} \leq TOL \) (elastic step; e.g., \( TOL = 10^{-8} \)):

\( \hat{\mathbf{e}}_{n+1} = 0; \) SET \( \mathbf{e}_{n+1} = \mathbf{e}_{n+1}^r, \mathbf{e}_n^p = \mathbf{e}_n^p; \) GO TO (iv) in Box 1.

Otherwise (plastic step): GO TO (e).

(e) Perform plastic correction; return mapping algorithm:

Solve nonlinear scalar equation for \( \hat{\mathbf{e}}_{n+1} \): use local Newton iteration

\[ F_{n+1}(\mathbf{e}_{n+1}) = [1 - (\phi_1)_n] \mathbf{e}_{n+1}^r : \mathbf{T}_{n+1} : \mathbf{e}_{n+1}^r - K^2(\mathbf{e}_{n+1}) = 0 \]

(f) Perform convergence check:

If \( |F_{n+1}(\hat{\mathbf{e}}_{n+1})| \leq TOL \) (e.g., \( 10^{-8} \)): then update

\( \hat{\mathbf{e}}_{n+1} = \hat{\mathbf{e}}_{n+1}^{(l)}, \mathbf{e}_{n+1} = \mathbf{e}_{n+1}^{(l)}, \mathbf{e}_n^p = (\mathbf{e}_n^{(l)})^p, \mathbf{e}_n^{(l)} = (\mathbf{e}_n^{(l)}); \) EXIT.

Otherwise: SET \( l = l + 1 \)

Compute derivative of \( F([\hat{\mathbf{e}}_{n+1}]) : DF([\hat{\mathbf{e}}_{n+1}]) \)

\[ (\hat{\mathbf{e}}_{n+1})^{(l+1)} = (\hat{\mathbf{e}}_{n+1})^{(l)} - DF([\hat{\mathbf{e}}_{n+1}]) ; \] GO TO (e).
Cantilever Composite Beam under Impact

Displacement at the free edge during impact

Damage index, representing the volume fraction of damaged fibers, at free and fixed edges during impact
Planar biaxial tests of cruciform shaped composite specimen were performed by Waas and Quek (1999) to gain insight into constitutive behavior of damage induced composite materials.

Numerical simulations of biaxial tests were carried out to examine whether the implemented computational model is able to predict the experimentally obtained response.

Composite specimen was loaded proportionally with the rate of 30 lb/sec (130 N/sec) in biaxial compression and tension in the ratio of 1:1.

Material properties, volume fraction of fibers, and aspect ratio of fibers were $E_m=3.0\text{GPa}$, $n_m=0.35$, $E_f=72\text{GPa}$, $n_f=0.17$, $\phi=0.5$, and $a=20.0$. Default DYNA3D shell element formulation based on Belyschko-Tsay theory was utilized for modeling the composite specimen.
Biaxial Test Simulations

See Figures 1 and 2.
Damage Contours During Biaxial Loading

† Figures show a sequence of damage contours during biaxial loading. Damage zone emanates from the edges of the cutout.

† Figure 2 of previous slide and above figures show damage initiation and evolution along the x- and y-axes around the cutout, which corresponds to Waas and Quek [1] ’s observations.

† Simulations are used for predicting the direction and the rate of damage propagation during the test.
### Four-Point Bend Simulation

- For progressive crush tests of composite tubes, flexural properties and the corresponding damage mechanisms of components must be characterized.
- Four-point bend test is commonly used for determining the flexural properties of high-strength composites.
- To assess the predictive capability of the computational model for simulating impact damage of composite structures, numerical four-point bend simulations were carried out.
- In order to avoid excessive indentation, or failure due to stress concentration directly under the loading nose, the loading nose in contact with the specimen must be sufficiently large.
- Rigid loading noses and rigid supports were modeled as the master surface and the set of nodes on the composite plate as the slave contact node set.
**Four-Point Bend Simulation**

*Figures show the maximum damage at surfaces located around loading noses.*

Sequence of damage contours and deformed shape during four-point bend impact

† Figures show the maximum damage at surfaces located around loading noses.
Time-history plots for the damage index at the contact surface near loading nose and at the center between loading nose and support

† Simulation will be performed with a geometrical trigger at the center of the specimen.
Parametric Study for Weibull Parameter $S_o$

Damage index, representing the volume fraction of perfectly bonded fibers, during four-point loading

* Influence of the value of Weibull parameter $S_o$, which is related to the interfacial strength, on the damage evolution in composite.

$\sigma_y$: yield stress of the matrix
Summary

- Emanating from a constitutive damage model for aligned fiber-reinforced composites, a micromechanical damage constitutive model for random fiber-reinforced composites was developed to perform impact simulation of random fiber-reinforced composites for automotive applications.

- Evolutionary interfacial debonding model and a crack-weakened model were subsequently employed in accordance with the Weibull's probability function to characterize the varying probability of fiber debonding.

- Micromechanics-based inter-fiber interaction formulation has been developed to model composites with high fiber volume fraction.

- Developed constitutive model is able to simulate mechanical behavior of new composite materials (e.g., P4) containing cracks of different sizes and orientations by adding the corresponding inclusion phases.
<table>
<thead>
<tr>
<th>Summary</th>
</tr>
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<tbody>
<tr>
<td>† Constitutive models were implemented into the nonlinear finite element code DYNA3D using a user-defined material subroutine to simulate the dynamic inelastic behavior and the progressive damage of the composite materials.</td>
</tr>
<tr>
<td>† Composite impact simulations were carried out to predict the experimentally obtained response during impact.</td>
</tr>
<tr>
<td>† Based on the numerical simulations, we can draw the conclusion that although more experimental work is needed to determine the damage parameters, the implementation of a new constitutive model into the finite element code DYNA3D has resulted in a promising numerical tool for the simulation of progressive damage in impacted composite structures.</td>
</tr>
</tbody>
</table>
Future Directions

† Develop laboratory experiments and procedures for characterizing basic damage mechanisms and monitoring the damage evolution during impact using nondestructive evaluation techniques.

† Conduct further assessment and experimental validation of the developed damage models to simulate composite crushing problems.
Fracture Mechanics Based Damage Modeling of RFC Composites

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21 March, 2000
Damage Modeling of RFC Composites

- Prone to failure by accumulation of damage and growth of cracks

- The failures have origins in microstructural defects such as
  - Micro-cracks in matrix material
  - Weak material interfaces
  - Voids and secondary crack surfaces

- Mechanical performance is directly related to resist failure by crack initiation and subsequent growth, i.e. the Fracture Toughness of the material
Damage Modeling of RFC Composites

- Build upon existing theories that account for the microstructure of the material
- Allow for the evolution of the microstructure by employing the governing physical laws
- Model the stress relaxation/softening with evolving microstructure
- Use traditional fracture mechanics based testing methods to calibrate the model
Properties of Micro-cracked Solids

- Several studies in literature focused on the prediction of properties of micro-cracked solids

- Studies take into account the micro-crack opening, frictional sliding, crack interaction

- Consider only a *stationary system* of micro-cracks. Examples are the
  - Self Consistent scheme
  - Differential Scheme
Existing Theories - Secant Moduli

- Differential Scheme Estimates (Hashin 1988)

\[ \rho = \frac{5}{8} \ln \frac{\nu}{\nu_e} + \frac{15}{64} \ln \left( \frac{1 - \nu_e}{1 - \nu} \right) + \frac{45}{128} \ln \left( \frac{1 + \nu_e}{1 + \nu} \right) + \frac{5}{128} \ln \left( \frac{3 - \nu_e}{3 - \nu} \right) \]

\[ E_e = E \left( \frac{\nu_e}{\nu} \right)^{10/9} \left( \frac{3 - \nu}{3 - \nu_e} \right)^{1/9} \]

- Self Consistent Estimates (Budiansky & O’Connell, 1976)

\[ E_e = E \left( 1 - \rho \frac{16(1 - \nu^2)(10 - 3\nu)}{45(2 - \nu)} \right) \]

\[ \nu_e = \nu \left( 1 - \rho \frac{16(1 - \nu^2)(3 - \nu)}{15(2 - \nu)} \right) \]
Evolving Microstructure

- During material failure, exiting micro-cracks grow in size, new micro-cracks are formed, and they subsequently interact and coalesce.

- Energy absorbed is determined by these failure mechanisms which affect the post-failure response.

- In computational modeling:
  - Need to estimate the tangent moduli $D_t$ for an evolving system of micro-cracks.
  - Capture stresses relaxation accompanying accumulation of damage.
Rate Constitutive Relations

• Constitutive relation for elastic material with micro-cracks distributed statistically uniformly

\[ \sigma = D_s (c, N) : \varepsilon \]

• Rate equations for evolving microstructure are obtained as

\[ \dot{\sigma} = D_s : \dot{\varepsilon} + \dot{D}_s : \varepsilon = D_e : \dot{\varepsilon} + \frac{\partial D_s}{\partial c} \dot{c} : \varepsilon + \frac{\partial D_s}{\partial N} \dot{N} : \varepsilon \]

• Constitutive description is complete by specifying
  – Micro-crack growth rate, \( \dot{c} \)
  – Micro-crack nucleation rate, \( \dot{N} \)
Experimental observations reveal

– Cracks grow at a fraction of Raliegh surface wave speed (Ravi-Chandar & Knauss 1984, Freund 1998)

\[ \dot{c} = \alpha(\sigma)v_R, \quad \alpha < 1.0 \quad v_s = \sqrt{\frac{E_e}{2\rho(1+\nu)}}, \quad v_R = v_s \frac{0.862 + 1.14v_e}{1 + v_e} \]

– Crack nucleation is governed as (Seaman et al., 1976)

\[ \dot{N} = \dot{N}_0 \exp\left(\frac{P - P_0}{P_1}\right), \quad P > P_0 \quad \dot{N} = 0, \quad P \leq P_0 \]
Micro-Crack Stability

- Micro-crack stability is governed by energy balance as specified by Griffith fracture theory.
- The stress state, normal and shear stresses, surrounding a crack tip governs the crack driving energy, $G$.
- Cracks grow when crack driving energy $G$ is greater than the fracture toughness of material, $\Gamma_R$. 

\[ \text{energy balance} \]

\[ \sigma \]

\[ \sigma \]
Stresses on a Micro-Crack

• Normal traction on a micro-crack

\[ \sigma_n = \sigma_{ij} n_i n_j \]

• Shear traction on a micro-crack

\[ \sigma_t = \left[ \sigma_{ij} n_j \sigma_{ik} n_k - (\sigma_{ij} n_i n_j)^2 \right]^{1/2} \]

• Frictional resistance stress are

\[ \sigma_r = \sigma_0 - \mu (\sigma_{ij} n_i n_j) \]
Micro-Crack Stability Criteria

Application of Griffith Criteria for a penny shaped micro-crack results in the following crack stability criteria (Addessio & Johnson 1990)

- Crack growth under tension \( (\sigma_n = \sigma_{ij} n_i n_j \geq 0) \)

\[
F_n(\sigma_{ij}, n_i, c) = \sigma_{ij} n_j n_k - \frac{\nu}{2} \left( \sigma_{ij} n_i n_j \right)^2 - \overline{K}/c \geq 0
\]

- Crack growth under compression \( (\sigma_n = \sigma_{ij} n_i n_j < 0) \)

\[
F_t(\sigma_{ij}, n_i, c) = (\sigma_t - \sigma_r)^2 - \overline{K}/c \geq 0
\]

\[
\overline{K} = \frac{\pi}{2} \left( \frac{2 - \nu}{1 - \nu} \right) \frac{\Gamma_R G}{c}
\]
Damage Surfaces

- Continuum level representation of micro-crack stability criteria is a *damage surface* and is obtained through the following volume average procedure (Addessio & Johnson 1990)

\[
\iiint_{\Omega} F(\sigma_{ij}, n_i, c) w(c, \Omega) \, dc d\Omega dV \geq 0
\]

- The resulting damage surface are defined in terms of the scalar parameters, p and q, defined as

\[
p = -\frac{1}{3} \sigma_{ii} \quad \text{and} \quad q = \left(\frac{3}{2} s_{ij} s_{ij}\right)^{1/2}, s_{ij} = \sigma_{ij} + p \delta_{ij}
\]
Damage Surfaces

(Addessio & Johnson 1990)

• In tension, \( p < 0 \), damage surface is obtained as

\[
q^2 + \frac{45}{4(5-v)} \left[ (2-v)p^2 - \frac{2K}{c} \right] = 0
\]

• In compression, \( p > 0 \), damage surface is obtained as

\[
q^2 - \frac{45}{3(3-2\mu^2)} \left[ \left( \mu p + \sigma_0 + \sqrt{\frac{\pi K}{c}} \right) + \frac{2K}{c} \right] = 0
\]
Damage Surfaces

Graphical plot of the damage surface in the (p,q,c) space
Damage Surfaces

Graphical plots of the damage surface in the (p,q) space for various fixed values of micro-crack length, c
Dissipative Nature of Material

• As cracks grow crack distribution evolves and the stiffness/compliance varies in time

\[ \dot{\varepsilon} = C_e : \dot{\sigma} + \dot{C}_e : \sigma \]

• Rate equation for a Maxwell solid

\[ \dot{\varepsilon} = \frac{\dot{\sigma}}{E} + \frac{\sigma}{\eta} \]

• Material behaves as a Maxwell solid with a variable relaxation time, \( \tau \)

\[ \tau = \frac{\eta}{E} = \frac{C_e}{\dot{C}_e} \]
## Material Point Constitutive Algorithm

1. Initialize: \( c_{n+1} = c_n, \varepsilon_{n+1} = \varepsilon_n + \Delta \varepsilon_{n+1} \)

2. Compute \( D_s(E_e, \nu_e, c_{n+1}, N_0) \) and \( \Delta \sigma = D_s : \Delta \varepsilon_{n+1} \)

3. Compute \( p_{n+1} \) and \( q_{n+1} \) and \( F_d(p_{n+1}, q_{n+1}, c_{n+1}) \)

4. If Damage Surface, \( F_d < 0 \). YES: EXIT

   ELSE: *Evolve micro-cracks*

5. Compute incremental micro-crack growth

   \[
   c_{n+1} = c_n + \Delta t \alpha(\sigma) \bigg|_n v \bigg|_{n+1} \]
Material Point Constitutive Algorithm

6. Update the Moduli $D_s$ using, $c_{n+1}$

7. Update stresses using incremental constitutive equation

$$\sigma_{n+1} = \sigma_n + D_e|_{n+1} : \Delta \varepsilon_n + \frac{\partial D_e|_{n+1}}{\partial c_{n+1}} (c_{n+1} - c_n) : \varepsilon_{n+1}$$

8. EXIT
## Material Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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</thead>
<tbody>
<tr>
<td>$E_0$</td>
<td>$4.4 \times 10^{11}$ Pa</td>
</tr>
<tr>
<td>$\nu_0$</td>
<td>0.16</td>
</tr>
<tr>
<td>$\rho$</td>
<td>3177 Kg/m$^3$</td>
</tr>
<tr>
<td>$\sigma_0$</td>
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<tr>
<td>$\Gamma_R$</td>
<td>10.13 N/m</td>
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<tr>
<td>$\mu$</td>
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</table>

<table>
<thead>
<tr>
<th>Parameter</th>
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<tbody>
<tr>
<td>$N_0$</td>
<td>$10^{11}$ m$^{-3}$</td>
</tr>
<tr>
<td>$c_0$</td>
<td>14 µm</td>
</tr>
<tr>
<td>$\rho_0$</td>
<td>$Nc_0^3 = 2.744 \times 10^{-4}$</td>
</tr>
</tbody>
</table>

Representative of SiC material
Uniaxial Strain Test

- Stress (Pa)
  - Tension
  - Compression

- % ε
  - 0
  - 2
  - 4
  - 6

- Speeds:
  - 250 s⁻¹
  - 2500 s⁻¹
  - 10,000 s⁻¹
Uniaxial Strain Tension

Load path in the (p,q,c) space

Initial Micro-Crack growth
Uniaxial Strain Compression

Load path in the \((p,q,c)\) space

- **Initiation of micro-crack growth**

\[
\begin{align*}
p / P_0 & \\
q / q_0 & \\
c & 
\end{align*}
\]
Softening – Uniaxial Strain Tension

Effect of superimposed hydrostatic pressure on subsequent stress softening

A – Stress path without superposed hydrostatic pressure

B – Stress path with superposed hydrostatic pressure after peak in curve A
Softening – Triaxial Strain Tension

Effect of superimposed hydrostatic pressure on subsequent stress softening

A – Stress path without superposed hydrostatic pressure

B – Stress path with superposed hydrostatic pressure after peak in curve A
Experimental Measurement of $\Gamma_R$

- The various energy dissipating damage mechanisms are expected to result in an R-curve like crack growth in RFCC’s (Gaggar & Broutman, 1975)

- Device a SEN or a DCB test specimen program to evaluate the R-Curve for the composite
Calibration of N and $c_0$

- Use the experimentally obtained P-u curve to calibrate the model.
- Numerically simulate the test to reproduce the P-u curve and obtain representative values for N and $c_0$. 
Delamination Modeling Using Cohesive Elements

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Delamination Failure of Composites

- Failure in RFC composites is observed to occur through delamination between plies in the composite.

- Delamination involves propagation of crack surfaces accompanied by energy dissipation.
Delamination - Cohesive Elements

- Energy dissipation during delamination occurs within the cohesive zone located ahead of propagating crack front.

- In computational modeling of delamination, cohesive elements are a powerful tool to model automatic failure initiation and subsequent propagation.

\[ \sigma \]

Cohesive Zone

Cohesive Element

σ
Cohesive Zone Potentials

- Cohesive Zone Potentials, $\phi$, are used to represent energy dissipation occurring during delamination

$$
\phi_{XN}(\Delta_n, \Delta_{t1}, \Delta_{t2}) = \Gamma_0 \left[ 1 - \left(1 + \frac{\Delta_n}{\delta_{cr}}\right) \exp\left(-\frac{\Delta_n}{\delta_{cr}}\right) \exp\left(-\frac{\Delta_{t1}^2 + \Delta_{t2}^2}{2\delta_{cr}^2}\right) \right]
$$

- Potentials allow for coupling of deformation between the normal and tangential modes of delamination
Cohesive Zone Traction

- Traction across crack faces are obtained as:

\[ T_n = - \frac{\partial \phi(\Delta_n, \Delta_t, \Delta_{t2})}{\partial \Delta_n}, \text{etc} \]

- Normal Traction \( T_n \) and Tangential Traction \( T_t \)

- Model parameters are:

\[ \Gamma_0 = \int_{0}^{\delta_{cr}} T_n(\Delta_n, \Delta_{t1}, \Delta_{t2}) \, d\Delta_n \]

\( \delta_{cr} \) - Critical opening displacement
Dynamic Crack growth in a DCB

- Cohesive element implemented in DYNA3D
- Verifies our implementation of cohesive element
Future Efforts

- Develop better approximations to micro-crack growth velocity based on analytical solutions for a penny-shaped crack under dynamic loading conditions.

- Consider effects of fibers and surrounding micro-cracks on energy release rate and corresponding damage surfaces:
  - Stang 1986 and Pijauder-Cabot & Bazant 1991
  - Effects of secondary tensile cracks in compression.

- Obtain a plane stress formulation for use with shell elements:
  - $\sigma_{33} = 0$, $\Delta \varepsilon_3$ in not kinematically specified.
Future Efforts

• Investigate into the reduced integrated versus the fully integrated shell formulations to be used with the model effectively
  – Belytschko-Tsay element
  – Hughes-Liu element

• Formalize the experimental program for the calibration of the model

• Simulate the quasi-static and dynamic RFC composite crush tube tests
Modeling of P4 Preform

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# P4 Microstructure Model

- Model currently simulates fiber deposition, only
- Complete process model should include preform compression, resin infusion and curing
- Model is used for linking of manufacturing process to resulting mechanical properties
- P4 fiber deposition simulation provides information for micro-mechanical model to account for characteristic distributions of preform and fiber properties
Fiber Deposition Process

10 fibers

50 fibers

100 fibers

460 fibers
Fiber Deposition Model

- Process variables used in simulation:
  - Spatial and orientation distributions
  - Fiber length, cross-section area, filamentization, flexibility, cross-section deformation
- Programmed deposition results in characteristic fiber property distributions
- Process variables ultimately determine distribution of effective material properties
Preform Analysis

- Fiber-fiber and fiber-resin contact areas indicate interaction between constituents
- Preform sectioning in longitudinal and through-thickness direction indicate effective lamina properties distributions, and representative volumes, and delamination paths
Preform Analysis

- Line representation is used for analysis of longitudinal fiber properties
- Free volume analysis is used for determination of characteristic resin volume distribution
Preform Measures

• Implemented measures:
  – Fiber-to-fiber contact area distribution
  – Fiber-to-resin contact area distribution
  – Fiber longitudinal shape distribution
  – Fiber cross-section distribution

• For random fiber deposition, fiber features vary in composite through-thickness direction
Fiber-to-Fiber Contact Area

- Fiber flexibility in axial and cross-section directions control extent of contact.
Fiber Shape Analysis - Wavelet Spectrum
Fiber Shape Analysis - Power Spectrum
Fiber Shape Analysis

- Plots are continuously updated during layup
- Fibres lying lowest in the layup bear the lowest frequency components (and fewer components in general)
- Fibers lying higher in the layup bear the highest frequencies since they must adjust to ever more complicated "substrates"
- Frequencies stabilize after certain thickness is reached
## Relevance to Material Modeling

- Fiber shape characterization is used for determination of equivalent fiber shape distributions in micro-mechanics-based model
- Fiber shape distribution and fiber-to-fiber/resin contact is used for solution of single fiber problem
- Material properties significantly vary in through-thickness direction and influence delamination
- Distribution of preform elastic properties can indicate representative volume of material